



Time, Value, Money:

Applications on the TI-83

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Classic Problems at a Glance using the TVM Solver

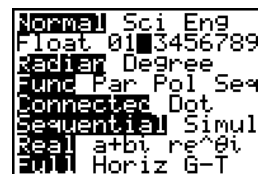
The table below illustrates the most common types of “classic” finance problems. The formulas are given for each calculation. A brief introduction to using the TVM Solver is followed by an example of each of the formulas in the table.

	Future Value	Present Value
Lump Sum	<ul style="list-style-type: none"> Compound Amount Factor $(1+i)^n$ 	<ul style="list-style-type: none"> Present Value Factor $\frac{1}{(1+i)^n} = (1+i)^{-n}$
Annuity	<ul style="list-style-type: none"> Annuity Compound Amount Factor $\frac{(1+i)^n - 1}{i}$ Sinking Fund Factor $\frac{i}{(1+i)^n - 1}$ 	<ul style="list-style-type: none"> Annuity Present Value Factor $\frac{1 - (1+i)^{-n}}{i}$ Capital Recovery Factor or Loan Repayment Factor $\frac{i}{1 - (1+i)^{-n}}$

Calculator Housekeeping Detail — Setting the Mode

- Press the **[MODE]** key (2B)[†] and change the Float to 2. This will cause decimal answers to display with 2 digits since most answers in finance are in dollars and cents. Highlight 2 on the second line and press **[ENTER]** to save your selection. (Figure 1)

(Figure 1)



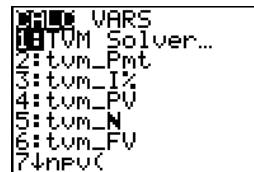
- Use **[2nd]** **[QUIT]** (2B) to return to the Home Screen.

[†] Refer to the section on Key Arrangement in Chapter 1 for an explanation of the key locator codes used in this manual.

Calculator Housekeeping Detail — Using the TVM Solver

- Press **[2nd]** **[FINANCE]** (5A). Choose **1:TVM Solver** from the CALC menu by pressing 1 or **[ENTER]**. (Figure 2)

(Figure 2)



```

1:TVM Solver...
2:tvm_Pmt
3:tvm_I%
4:tvm_PV
5:tvm_N
6:tvm_FV
7:↓nPV
  
```

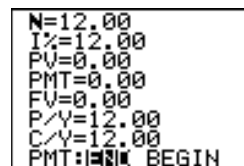
The screen that appears will be similar to Figure 3 but will probably have different values.

Interest is simple or compounded. In the TVM Solver, the number of compounding periods per year (C/Y) is set to 1 for simple interest; otherwise it indicates the number of compounding periods per year. For continuous compounding, set C/Y to a very large number, e.g., 1 E 11. (See *Chapter 6, Continuous Compounding* for details.)

- When using the TVM Solver, enter values for each of the variables:

N	Number of payment periods
I%	Annual interest rate
PV	Present Value
PMT	Payment Amount
FV	Future Value
P/Y	Number of payment periods per year
C/Y	Number of compounding periods per year
PMT:	Indicates whether payments are made at the end or beginning of each payment period. (Figure 3)

(Figure 3)



```

N=12.00
I%=12.00
PV=0.00
PMT=0.00
FV=0.00
P/Y=12.00
C/Y=12.00
PMT: END BEGIN
  
```

Note: Store C/Y after storing P/Y because a change in P/Y always resets C/Y.

- Enter the number of years for N, I% as a percent (not as a decimal). Enter the money invested as a negative number (cash outflows are considered negative, while cash inflows are considered positive). Enter the payment (PMT) as 0, and the payments per year (P/Y) as 1.
- To solve, move the cursor to the variable for which you want to solve. Press **[ALPHA]** **[SOLVE]** (10E) to calculate this value. A small shaded box to the left of the variable indicates a solution.

If you change a value on the screen, you will lose the shaded box and must solve again for a new value.

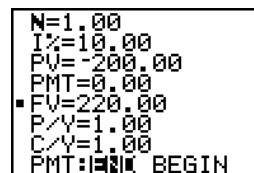
Compound Amount Factor $(1+i)^n$

Example 1:

What can an investor expect to receive at the end of a year if he deposits \$200 in a bank giving 10% interest?

- Press **[2nd]** **[FINANCE]** (5A) and choose **1:TVM Solver** from the CALC menu.
- As in step 3 above, enter N = 1, I% = 10, PV = -200, PMT = 0, P/Y = 1 and C/Y = 1. Leave PMT: set to END.
- Place the cursor on the Future Value (FV), then press **[ALPHA]** **[SOLVE]** (10E).

(Figure 4)



```

N=1.00
I%=10.00
PV=-200.00
PMT=0.00
FV=220.00
P/Y=1.00
C/Y=1.00
PMT: END BEGIN
  
```

The future value is \$220.00. (Figure 4)

Example 2:

How long will it take a dollar to double at 8% compounded semiannually?

1. Press **[2nd] [FINANCE]** and choose **1:TVM Solver**
2. In this example, I% = 8, PV = -1, PMT = 0, FV = 2, P/Y = 2 and C/Y = 2. To enter these values, input **[8] [↓] [(-)] 1 [↓] 0 [↓] 2 [↓] 2 [↓] 2**.
3. Place the cursor on the N and then press **[ALPHA] [SOLVE] (1OE)** to find the number of periods. (Figure 5)

(Figure 5)

```

N=17.67
I%=8.00
PV=-1.00
PMT=0.00
FV=2.00
P/Y=2.00
C/Y=2.00
PMT:0.00 BEGIN
  
```

It takes 17.67 payment periods, or 17.67 half-years. That is about 9 years.

(Figure 6)

```

N=17.67
I%=8.00
PV=-1.00
PMT=0.00
FV=2.00
P/Y=1.00
C/Y=2.00
PMT:0.00 BEGIN
  
```

4. By changing P/Y to 1, we can find the answer in years. Set P/Y = 1 and C/Y = 2. (Figure 6)
5. Move the cursor to the first line, N= and press **[ALPHA] [SOLVE] (1OE)**. (Figure 7)

(Figure 7)

```

N=8.84
I%=8.00
PV=-1.00
PMT=0.00
FV=2.00
P/Y=1.00
C/Y=2.00
PMT:0.00 BEGIN
  
```

Note: N = 8.84 years since the number of payments per year is set to 1.

Present Value Factor $\frac{1}{(1+i)^n} = (1+i)^{-n}$

Example 3:

What amount needs to be deposited in an investment that yields 5% so that it will be worth \$600 at year end?

(Figure 8)

```

N=1.00
I%=5.00
PV=-571.43
PMT=0.00
FV=600.00
P/Y=1.00
C/Y=1.00
PMT:0.00 BEGIN
  
```

1. Input 1 for N because the number of payment periods is one, 5 for the interest rate (I%), 0 as the payment (PMT), 600 as the future value (FV), and 1 for both payments per year (P/Y), and compoundings per year (C/Y).
2. Place the cursor on PV and press **[ALPHA] [SOLVE] (1OE)** to find the present value. (Figure 8)

One needs to deposit \$571.43 in an investment that yields 5% so that it will be worth \$600 at year end.

Annuity Compound Amount Factor $\frac{(1+i)^n - 1}{i}$

Example 4:

What can a finance teacher withdraw from her account if she saves \$1,000 a year for 3 years at 5% interest?

(Figure 9)

```

N=3.00
I%=5.00
PV=0.00
PMT=-1000.00
FV=3152.50
P/Y=1.00
C/Y=1.00
PMT:0.00 BEGIN
  
```

1. Input 3 for N because there are 3 payment periods, 5 for the interest rate (I%), 0 as the present value (PV), -1000 for the payment (PMT), 1 for payments per year (P/Y), and 1 for compounding per year (C/Y). Note that PMT is negative because the teacher is paying out \$1,000 every year.
2. Place the cursor on FV and press **[ALPHA] [SOLVE] (1OE)** to find the future value. (Figure 9)

In three years, the finance teacher can withdraw \$3,152.50.

Example 5:

A biologist deposits \$1,000 in her account at the end of each quarter for 10 years. How much money does she have at the end of 10 years if the bank pays 6% interest compounded quarterly?

The number of payment periods is 40 because she is making deposits four times a year for 10 years. Enter 0 for the present value (PV). The PMT must be entered as a negative number because it represents an outflow from the pocket of the biologist. The payments per year (P/Y) and the compounding per year (C/Y) are both 4 for quarterly compounding. The payments are at the end of each quarter, so highlight END rather than BEGIN in the last line of the **TVM Solver**. (Figure 10)

(Figure 10)

```
N=40.00
I%=6.00
PV=0.00
PMT=-1000.00
FV=0.00
P/Y=4.00
C/Y=4.00
PMT: [END] BEGIN
```

1. Input the information, then move the cursor to FV and press **[ALPHA]** **[SOLVE]** (10E) to find the present value. (Figure 11)

At the end of 10 years, the biologist has \$54,267.89.

(Figure 11)

```
N=40.00
I%=6.00
PV=0.00
PMT=-1000.00
FV=54267.89
P/Y=4.00
C/Y=4.00
PMT: [END] BEGIN
```

Example 6:

The biologist in the last exercise decided that she wanted to have \$54,267.89 in her account in 10 years and she found a bank which compounded monthly at 6%. What are her monthly payments to achieve her goal of \$54,267.89?

1. Change the number of payments (N) to 120 because the payments are now monthly for 10 years, and change the P/Y and C/Y to 12 because both now occur monthly.
2. Place the cursor on the payment (PMT) and press **[ALPHA]** **[SOLVE]** (10E) to find the amount of the payments. (Figure 12)

(Figure 12)

```
N=120.00
I%=6.00
PV=0.00
PMT=-331.15
FV=54267.89
P/Y=12.00
C/Y=12.00
PMT: [END] BEGIN
```

The biologist must make monthly payments of \$331.15 to achieve her goal of \$54,267.89.

Notice that 120 payments at \$331.15 amounts to \$39,738 out of the pocket versus 40 payments of \$1,000 each which amounts to \$40,000.

Annuity Present Value Factor $\frac{1 - (1 + i)^{-n}}{i}$

Example 7:

What would grandparents have to spend to establish a payout of \$2,000 per year for 3 years to their grandson at 5% interest?

1. Let the number of payments (N) be 3 because the payout is annual for 3 years. The interest rate (I%) is 5, the payment (PMT) of 2000 is positive because it is coming to the grandson. Enter 0 for the future value (FV), 1 for payments per year (P/Y) and 1 for compounding per year (C/Y).
2. Place the cursor on the present value (PV) and press **[ALPHA]** **[SOLVE]** (10E) to find the present value. (Figure 13)

The grandparents should invest \$5,446.50.

(Figure 13)

```
N=3.00
I%=5.00
PV=-5446.50
PMT=2000.00
FV=0.00
P/Y=1.00
C/Y=1.00
PMT: [END] BEGIN
```

The present value is negative on the calculator screen because that is the amount that the grandparents must pay out to establish the grandson's annuity.

Sinking Fund Factor $\frac{i}{(1+i)^n - 1}$

Example 8:

At the end of 3 years, a business person needs to have \$4,000 available to buy a new computer. How much money must he deposit each year at 5% interest?

1. Input 3 for the number of payments (N), 5 for the interest rate (I%), 0 for the present value (PV), 4000 as a positive value for the future value (FV), 1 for the payments per year (P/Y), and 1 for the compounding per year (C/Y).
2. Place the cursor on the payment (PMT) and press **[ALPHA]** **[SOLVE]** (10E) to find the annual payment. (Figure 14)

The annual payment is \$1,268.83.

The annual payment is negative on the calculator screen because it represents the amount that the business person must pay out every year.

(Figure 14)

```
N=3.00
I%=5.00
PV=0.00
PMT=-1268.83
FV=4000.00
P/Y=1.00
C/Y=1.00
PMT: [ALPHA] BEGIN
```

Capital Recovery Factor or Loan Repayment Factor $\frac{i}{1 - (1+i)^{-n}}$

Example 9:

What monthly payment would a college senior make to pay off a used car loan of \$2,000 at 12% by the end of the year?

1. Because there are monthly payments for a year, enter 12 for the number of payments (N), enter 12 for the interest rate (I%), 2000 for the present value (PV), 0 for the future value (FV), 12 for the payments per year (P/Y), and 12 for the compounding per year (C/Y). Note that (PV) is entered as a positive 2000 because the student has the money in hand.
2. Place the cursor on the payment (PMT) and press **[ALPHA]** **[SOLVE]** (10E) to find the monthly payment. (Figure 15)

The monthly payment will be \$177.70.

The monthly payment is negative on the calculator screen because the amount represents the amount that the college senior must pay out every month.

(Figure 15)

```
N=12.00
I%=12.00
PV=2000.00
PMT=-177.70
FV=0.00
P/Y=12.00
C/Y=12.00
PMT: [ALPHA] BEGIN
```

Depreciation

Depreciation of an asset allows periodic allocation of the cost of the asset. Tax law and accounting students use many methods for assigning the cost of an asset to the period during which it is used.

Straight Line Depreciation

Straight line depreciation is the simplest method and uses the basis of an asset and the useful life of the asset to assign equal depreciation to each period.

Example 1:

XYZ corporation wishes to depreciate a \$1,000 printer over its 5-year life using straight line depreciation. Calculate the values and the new basis for each year.

Let N = Useful life of asset in years

B = Basis of the asset

S = Salvage value

TD = Total depreciation allowed

Total depreciation allowed on a item is

$$TD = B - S$$

In these examples, salvage is assumed to be zero. For straight line depreciation

$$TD = 1000 - 0 = 1000$$

$$N = 5$$

$$\text{Periodic (annual depreciation)} = 1000/5$$

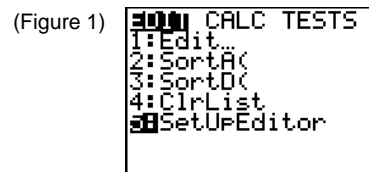
The adjusted basis $B(Y)$ at the end of the year Y is

$$B(Y) = 1000 (1 - Y \cdot (1/N))$$

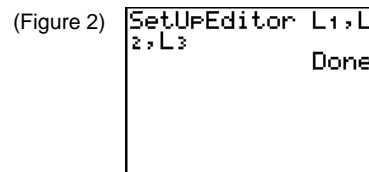
Straight line depreciation assigns $1/5$ of the basis value to each of the 5 years.

In this example, the list feature of the calculator will be used to construct a depreciation table.

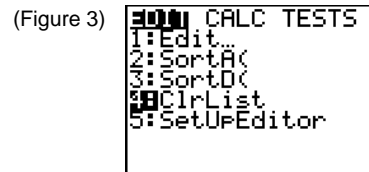
1. Press the **[STAT]** key (3C)[†] and choose **5:SetUpEditor** from the EDIT menu.
This will paste **SetUpEditor** on the Home Screen. (Figure 1)



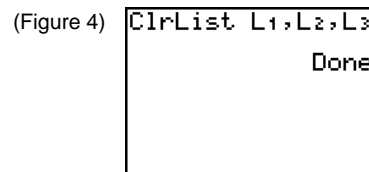
2. Type **[2nd]** **[L₁]** (9B) **,** **[2nd]** **[L₂]** (9C) **,** **[2nd]** **[L₃]** (9D) **[ENTER]**. The calculator will respond **Done**. (Figure 2)



3. Press the **[STAT]** key (3C) and choose **4:ClrList** from the EDIT menu to paste **ClrList** on the Home Screen. (Figure 3)

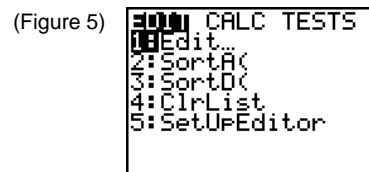


4. Type **[2nd]** **[L₁]** (9B) **,** **[2nd]** **[L₂]** (9C) **,** **[2nd]** **[L₃]** (9D) **[ENTER]**. The calculator will respond **Done**. (Figure 4)

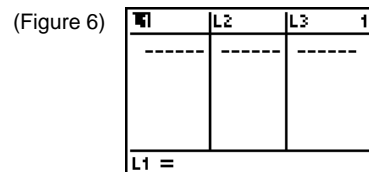


L₁ will store the numbers for the years.

5. Press the **[STAT]** key (3C) and choose **1:Edit** from the EDIT menu. (Figure 5)

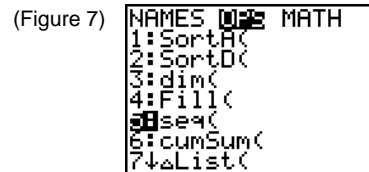


6. When the lists appear, move the cursor to the top of the column so that L₁ is highlighted and press **[ENTER]**. (Figure 6)

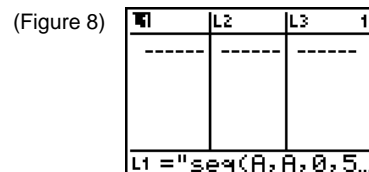


7. Press the quotes key, **[ALPHA]** **["]** (9E). Using quotes will cause the list to act like a spreadsheet.

8. Press **[2nd]** **[LIST]** (3C) and choose **5:seq(** from the OPS menu. (Figure 7)
The syntax for a sequence is **seq(expression,variable,begin,end)**.



9. Complete the expression so that L₁ contains **"seq(A,A,0,5)"**. (Figure 8)



[†] Refer to the section on Key Arrangement in Chapter 1 for an explanation of the key locator codes used in this manual.

The second column of the table will be the annual adjusted bases.

10. In L_2 enter “seq(1000(1-Y*(1/5)),Y,0,5)”. (Figure 9)

The last column, L_3 , will display the annual depreciation, which is the difference between the adjusted bases for consecutive years. Note that no depreciation is allowed for year 0. An easy way to set up L_3 is to use the **augment** and ΔList functions. For an explanation of these functions, see the Calculator Housekeeping Detail section that follows this example.

11. Enter “augment({0}, $\Delta\text{List}(L_2)$)” for L_3 . (Figure 10)

The **augment** function is accessed by pressing $\boxed{2\text{nd}} \boxed{[\text{LIST}]} (3C)$ and choosing

9:augment(from the OPS menu. ΔList is also found in the OPS menu.

(Figure 9)

L1	#	L2	L3	2
0				
1				
2				
3				
4				
5				
L2 = seq(1000(1-Y*(1/5)),Y,0,5)"				

(Figure 10)

L1	#	L2	#	L3	#	3
0		1000		0		
1		800		-200		
2		600		-200		
3		400		-200		
4		200		-200		
5		0		-200		
L3 = ..., $\Delta\text{List}(L_2)$ "						

Calculator Housekeeping Detail

The **augment** (list1, list2) command concatenates list1 and list2, creating a new list with the elements of list1 followed by the elements in list2. For example, if $L_1 = \{1,2,3\}$ and $L_2 = \{4,5\}$ then **augment** (L_1, L_2) would produce $\{1,2,3,4,5\}$.

1. Press $\boxed{2\text{nd}} \boxed{[\text{QUIT}]} (2B)$ to return to the Home Screen.
2. Type $\boxed{2\text{nd}} \boxed{[\{]} (6C) 1 \boxed{,} 2 \boxed{,} 3 \boxed{2\text{nd}} \boxed{[\}] (6D) \boxed{[\text{STO} \blacktriangleright]} (9A) \boxed{2\text{nd}} \boxed{[L_1]} (9B) \boxed{[\text{ENTER}]}$ to store $\{1,2,3\}$ in L_1 .
3. Type $\boxed{2\text{nd}} \boxed{[\{]} (6C) 4 \boxed{,} 5 \boxed{2\text{nd}} \boxed{[\}] (6D) \boxed{[\text{STO} \blacktriangleright]} (9A) \boxed{2\text{nd}} \boxed{[L_2]} (9C) \boxed{[\text{ENTER}]}$ to store $\{4,5\}$ in L_2 . (Figure 11)
4. Press $\boxed{2\text{nd}} \boxed{[\text{LIST}]} (3C)$ and choose **9:augment(** from the OPS menu to paste the function on the Home Screen. (Figure 12)
5. Type $\boxed{2\text{nd}} \boxed{[L_1]} \boxed{,} \boxed{2\text{nd}} \boxed{[L_2]} \boxed{[\text{ENTER}]}$. The result is a list containing $\{1,2,3,4,5\}$. (Figures 13 and 14)

(Figure 11)

(1,2,3)→L1
(1.00 2.00 3.00)
(4,5)→L2
(4.00 5.00)

(Figure 12)

NAMES OPS MATH
3:dim(
4:Fill(
5:seq(
6:cumSum(
7: $\Delta\text{List}($
8:Select(
9:augment(

(Figure 13)

(1,2,3)→L1
(1.00 2.00 3.00)
(4,5)→L2
(4.00 5.00)
augment(L1,L2)
(1.00 2.00 3.00...

(Figure 14)

(1,2,3)→L1
(1.00 2.00 3.00)
(4,5)→L2
(4.00 5.00)
augment(L1,L2)
...3.00 4.00 5.00)

Another interesting operation on the **OPS** menu under the $\boxed{2\text{nd}} \boxed{[\text{LIST}]} (3C)$ key is the $\Delta\text{List}(\text{listname})$ command. This operation creates a new list in which each element is the difference of successive elements of listname.

$$\Delta\text{List}(L_1) = \{L_1(2) - L_1(1), L_1(3) - L_1(2), \text{etc.}\}$$

$\Delta\text{List}(\{1,4,6,2\})$ yields the list $\{3,2,-4\}$. The new list will always have one less element in it than the original.

1. Press **2nd** [LIST] (3C) and choose **7:ΔList(** from the OPS menu. (Figure 15)

(Figure 15)

```

NAMES 0:28 MATH
1:SortA(
2:SortD(
3:dim(
4:Fill(
5:seq(
6:cumSum(
7:ΔList(

```

2. Complete the command by typing **2nd** [{] 1 [] 4 [] 6 [] 2 **2nd** [}] [] **ENTER**. (Figures 16 and 17)

(Figure 16)

```

ΔList({1,4,6,2})
{3.00 2.00 -4.00...

```

(Figure 17)

```

ΔList({1,4,6,2})
...00 2.00 -4.00}

```

In the earlier straight line depreciation example, the third column of the depreciation table showed the annual depreciation for each of the years 0 to 5. L_3 was generated by the expression.

$L_3 = \text{"augment } \{0\}, \Delta\text{List}(L_2) \text{"}$.

Observe that L_3 is really a list of the differences of the bases in L_2 except for the first element, 0, which is the depreciation for year 0.

L_3 is a list of the differences in the annual bases. $L_2(2) - L_2(1)$ is the depreciation allowed in year 1. $L_2(3) - L_2(2)$ is the depreciation allowed in year 2, etc. $\Delta\text{List}(L_2)$ does this calculation automatically and was used to calculate the depreciation for years 1 through 5. The depreciation for year 0 is 0. (Figure 18)

(Figure 18)

L1 #	L2 #	L3 #
0	1000	0
1	800	-200
2	600	-200
3	400	-200
4	200	-200
5	0	-200

$L_3 = \text{"augment } \{0\}, \Delta\text{List}(L_2) \text{"}$

Sum of the Digits Method of Depreciation

A classic depreciation technique is called the sum of the digits method and computes a different fractional depreciation for each year. The denominator of each fraction is the sum of the digits from 1 to N where N is the number of years in the life of the asset. The numerator is $N - Y + 1$ where Y is the period number.

Example 2:

Show a depreciation table for a sum of the digits method for 5 years on a \$1,000 printer.

The depreciation table will consist of 4 columns. L_1 is the year.

1. Enter **"seq(Y,Y,1,5)"** for L_1 .

L_2 is the fraction of the basis taken as depreciation for each year. The numerator of the fraction is $5 - Y + 1$ and the denominator is the sum of the digits 1 through 5.

- Enter “ $(1/\text{sum}(L_1))*(5-L_1+1)$ ” for L_2 . The **sum** function is accessed by pressing $\boxed{2\text{nd}} \boxed{[\text{LIST}]} (3C)$ and choosing **5:sum(** from the MATH menu.
(Figures 19 and 20)

(Figure 19)

L1	#	L3	#
1.00	.33	333.33	
2.00	.27	266.67	
3.00	.20	200.00	
4.00	.13	133.33	
5.00	.07	66.67	
-----	-----	-----	
L2 = "(1/sum(L1))*			

(Figure 20)

L1	#	L3	#
1.00	.33	333.33	
2.00	.27	266.67	
3.00	.20	200.00	
4.00	.13	133.33	
5.00	.07	66.67	
-----	-----	-----	
L2 = "...)*(5-L1+1)"			

L_3 is the depreciation, the original basis multiplied by the factor in L_2 .

- Enter “ L_2*1000 ” for L_3 . (Figure 21)

(Figure 21)

L1	L2	#	L3	#
1.00	.33		333.33	
2.00	.27		266.67	
3.00	.20		200.00	
4.00	.13		133.33	
5.00	.07		66.67	
-----	-----		-----	
L3 = "L2*1000"				

L_4 gives the basis at the end of each year and equals the original basis less the depreciation already taken.

- Enter “ $1000-\text{cumSum}(L_3)$ ” for L_4 . The **cumSum** command can be entered by pressing $\boxed{2\text{nd}} \boxed{[\text{LIST}]} (3C)$ and choosing **6:cumSum(** from the OPS menu.
(Figure 22)

(Figure 22)

L2	#	L3	#	L4	#
.33		333.33		666.67	
.27		266.67		400.00	
.20		200.00		200.00	
.13		133.33		66.67	
.07		66.67		0.00	
-----		-----		-----	
L4 = "1000-cumSum(

Double Declining Balance Depreciation

This depreciation method is allowed by the tax code and gives a larger depreciation in the early years of an asset. Unlike the straight line and the sum of the digits methods, both of which use the original basis to calculate the depreciation each year, the double declining balance uses a fixed percentage of the prior year's basis to calculate depreciation. The percentage rate is $2/N$ where N is the life of the asset. With this method, the basis never becomes zero. Consequently, it is standard practice to switch to another depreciation method as the basis decreases. Usually the taxpayer will convert to the straight line method when the annual depreciation from the declining balance becomes less than the straight line.

For example, if the life of an asset is 5 years, straight line depreciation allows $1/5$ or 20% of the basis as depreciation each year. Thus, a \$1,000 basis depreciates \$200 per year. The double declining balance method allows $2/5$ or 40%, double the straight line rate, of the current basis each year. In this example,

40% of \$1000 = \$400 in year 1

40% of \$600 = \$240 in year 2

40% of \$360 = \$144 in year 3

The double declining balance method relies on the new basis each year. This calculation is similar to finding compound interest.

Year	Basis
0	1000
1	$1000(1-.4)$
2	$(1000(1-.4))(1-.4) = 1000(1-.4)^2$
3	$((1000(1-.4))(1-.4))(1-.4) = 1000(1-.4)^3$

Note if the life had been 8 years, then straight line depreciation would allow only 12.5% of the original basis per year while the double declining balance would allow 25%.

Example 3:

Calculate double declining balance depreciation for an item with useful life of 8 years and a basis of \$1,000.

1. Store the years in L1: $L_1 = \text{"seq(Y,Y,0,8)"}$. (Figure 23)

(Figure 23)

L1	#	L2	L3	1
0.00		-----	-----	
1.00				
2.00				
3.00				
4.00				
5.00				
6.00				
L1 = "seq(Y,Y,0,8)"				

2. L_2 is the declining balance: $L_2 = \text{"1000(1-.25)^L1"}$. (Figures 24 and 25)

(Figure 24)

L1	#	L2	L3	2
0.00		1000.0	-----	
1.00		750.00		
2.00		562.50		
3.00		421.88		
4.00		316.41		
5.00		237.30		
6.00		177.98		
L2 = "1000(1-.25)^L1"				

(Figure 25)

L1	#	L2	L3	2
0.00		1000.0	-----	
1.00		750.00		
2.00		562.50		
3.00		421.88		
4.00		316.41		
5.00		237.30		
6.00		177.98		
L2 = ... (1-.25)^L1 "				

3. L_3 is the depreciation allowed: $L_3 = \text{"augment({0}, \Delta List(L_2))"}$. (Figures 26 and 27)

The **augment** function can be found by pressing **[2nd] [LIST] (3C)** and choosing **9:augment(** from the OPS menu. **ΔList** is also located in the OPS menu.

(Figure 26)

L1	#	L2	#	L3	#	3
0.00		1000.0		0.00		
1.00		750.00		-250.0		
2.00		562.50		-187.5		
3.00		421.88		-140.6		
4.00		316.41		-105.5		
5.00		237.30		-79.10		
6.00		177.98		-59.33		
L3 = "augment({0},						

(Figure 27)

L1	#	L2	#	L3	#	3
0.00		1000.0		0.00		
1.00		750.00		-250.0		
2.00		562.50		-187.5		
3.00		421.88		-140.6		
4.00		316.41		-105.5		
5.00		237.30		-79.10		
6.00		177.98		-59.33		
L3 = ... (ΔList(L2))"						