Review Sheet for AP Calculus Test on Sections 4.1 to 4.3

Name ______________________

A Calculator MAY NOT be used on this part of this Review Sheet

1. Find the absolute extrema of $y = 3x^{\frac{2}{3}} - x^2$ on $[-1, 1]$.

$y(-1) = 3(-1)^{2/3} - (-1)^2 = \boxed{2}$

$y(1) = 3(1)^{2/3} - (1)^2 = \boxed{2}$

$y' = 2x^{-\frac{1}{3}} - 2x$

$0 = 2x^{-\frac{1}{3}}\left[1 - x^{2/3}\right]$

$\boxed{x = 0, 1}$

$\Rightarrow y(0) = 3(0)^{2/3} - (0)^2 = \boxed{0}$

Abs Max of 2 at $x = \pm 1$

Abs Min of 0 at $x = 0$

2. Find the value of c guaranteed by the Mean Value Theorem for $y(x) = x^3 - x^2 - 2x$ on $[-1, 1]$.

$y'(x) = 3x^2 - 2x - 2$

$3x^2 - 2x - 2 = \dfrac{f(1) - f(-1)}{1 - (-1)}$

$3x^2 - 2x - 2 = \dfrac{-2 - 0}{2}$

$3x^2 - 2x - 2 = -1$

$3x^2 - 2x - 1 = 0$

$\Rightarrow (3x + 1)(x - 1) = 0$

$\boxed{x = -\frac{1}{3} \quad x = 1}$

3. Use the Second Derivative Test to find and label all relative extrema of $f(x) = 4x^3 - 2x^2$.

$f'(x) = 12x^2 - 4x$

$0 = 4x(3x - 1)$

$\boxed{x = 0, \frac{1}{3}}$

$f''(x) = 24x - 4$

$f''(0) = -4 < 0 \Longrightarrow$ Rel Max at $(0, 0)$

$f''\left(\frac{1}{3}\right) = 4 > 0 \Longrightarrow$ Rel Min at $\left(\frac{1}{3}, -\frac{2}{27}\right)$

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4. Can Rolle's Theorem be applied to $f(x) = x^4 - 2x^2$ on $[-2, 2]$? If so, find the value of c for which $f'(c) = 0$. If it cannot be applied, tell why.

Rolle's Thm DOES Apply.

f(x) is diff on (−2, 2)

f(x) is cont on [−2, 2]

$f(-2) = f(2) = 8$

$f'(x) = 4x^3 - 4x$

$0 = 4x(x^2 - 1)$

$\boxed{x = 0, \pm 1}$

5. Let $y'(x) = x^3 - 4x^2 + 3x$

a. Where do the extrema of y occur?

$y' = x(x-3)(x-1)$

$x = 0, 3, 1$

y′: − | + | − | +

y: dec 0 inc 1 dec 3 inc

Rel Max at x = 1

Rel Min at x = 0, 3

b. Find the intervals on which f is increasing & decreasing.

Inc: (0, 1), (3, ∞)

Dec: (−∞, 0) and (1, 3)

c. Find where f is concave up and concave down.

$y'' = 3x^2 - 8x + 3$

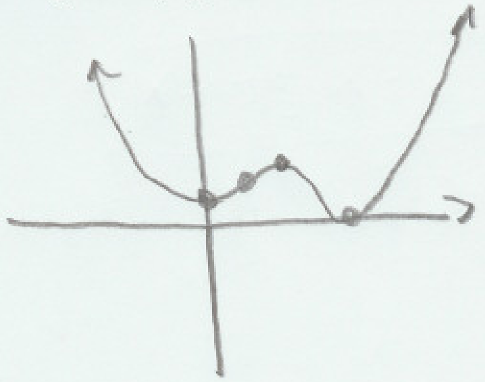
$0 = 3x^2 - 8x + 3$

$x = .451, 2.215$

y″: + | − | +

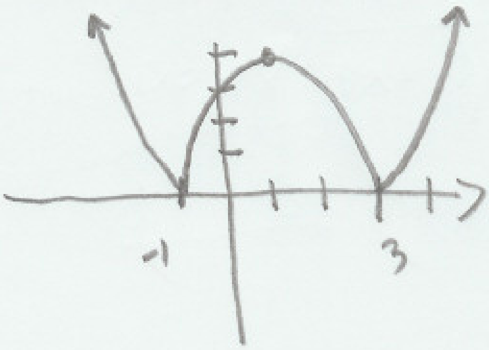
y: ccup .451 ccdn 2.215 ccup.

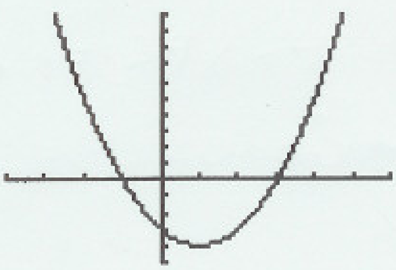
d. Sketch a possible graph for f.



6. Consider $f(x) = x^2 - 2x - 3$ sketched at right.

a. Sketch the graph of $y = |f(x)|$, below





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6. Continued

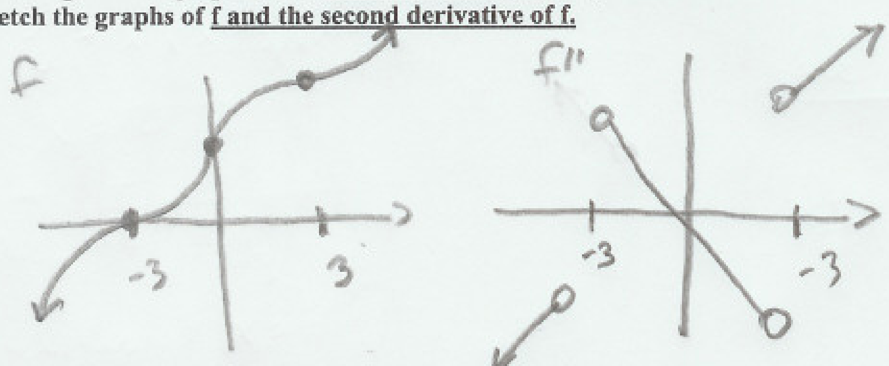
b. For $y = |f(x)|$ find the relative extrema and points of inflection. Why does $f'(3)$ NOT exist?

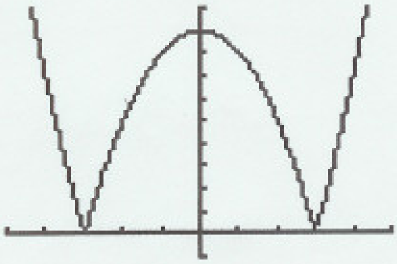
Rel Min in $(-1,0)$ and $(3,0)$

Rel Max $(0,4)$

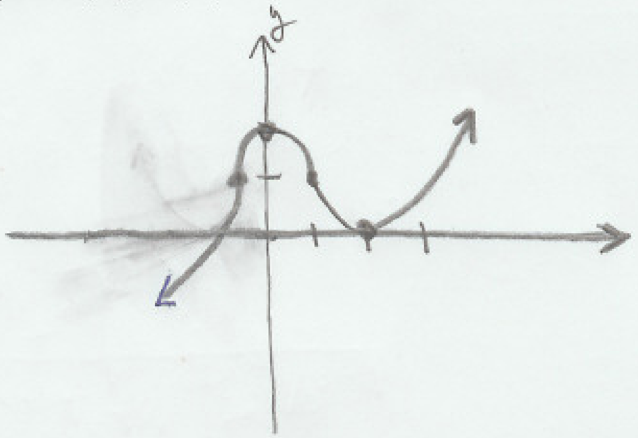
$f'(3)$ does not exist because the derivatives from left and right at $x=3$ are NOT equal.

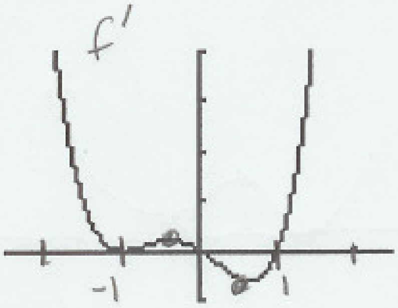
7. At right is the graph of the derivative of f. In the space below sketch the graphs of f and the second derivative of f.





8. At right is the graph of the derivative of f. Sketch a possible graph of $f(x)$, if $f(2) = 0$.





a. For what values of x does f have a relative max; relative min; points of inflection? On what intervals is $f(x)$ concave up?

① f' changes from pos to neg at $x=0 \Rightarrow$ Rel Max at $x=0$

② f' changes from neg to pos at $x=1 \Rightarrow$ Rel Min at $x=1$

③ POI's occur where f' changes from inc to dec or dec to inc. This happens at $x=-\frac{1}{2}$, $x=\frac{1}{2}$, and $x=-1$,

④ $f(x)$ is cc up on $(-1,-\frac{1}{2})$ and $(\frac{1}{2},\infty)$
$f(x)$ is cc dn on $(-\infty,-1)$ and $(-\frac{1}{2},\frac{1}{2})$

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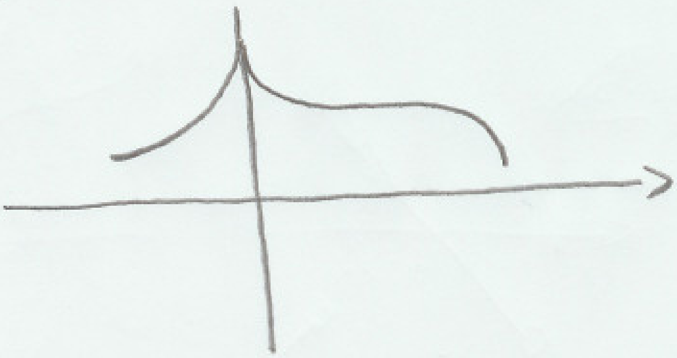
9. A function f is continuous on its domain [-2 , 4]. $f(-2)=5$, $f(4)=1$, and f′ and f″ have the following properties:

x	-2 < x < 0	x = 0	0 < x < 2	x = 2	2 < x < 4
f′	+	DNE	-	0	-
f″	+	DNE	+	0	-

a. Find where all absolute extrema of f occur. f has has an abs max at x = 0 and an abs min of 1 at x = 4. We can't determine f(0).

b. Find where the points of inflection of f occur.
f has a POI at x = 2 since f″ changes from pos to neg here.

c. Sketch a possible graph of f



10. Analyze and sketch: $y = x e^{x}$.

$\boxed{y' = e^x + e^x \cdot x}$

$0 = e^x(1+x)$

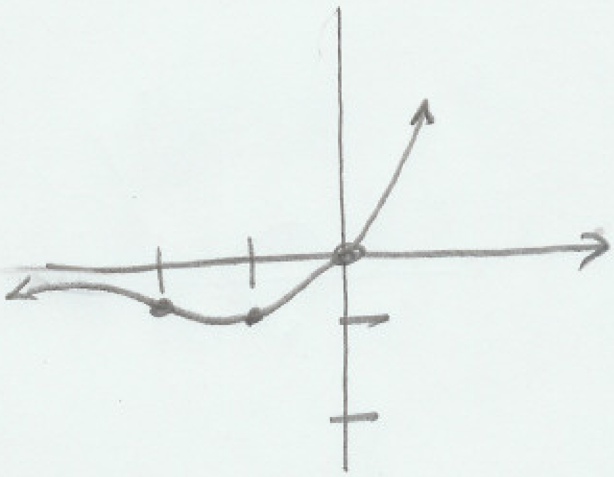
$\boxed{x = -1}$

$y'' = e^x(1+x) + e^x \cdot 1$

$y'' = e^x(1+x+1)$

$0 = (x+2)e^x$

$\boxed{x = -2}$



y': − | +
y: dec −1 inc.

y'': − | +
y: ccdn −2 ccup

Rel min at $\left(-1, -\frac{1}{e}\right)$

POI at $\left(-2, -\frac{2}{e^2}\right)$

$-\frac{1}{e} \approx -.368$

$-\frac{2}{e^2} \approx -.27$

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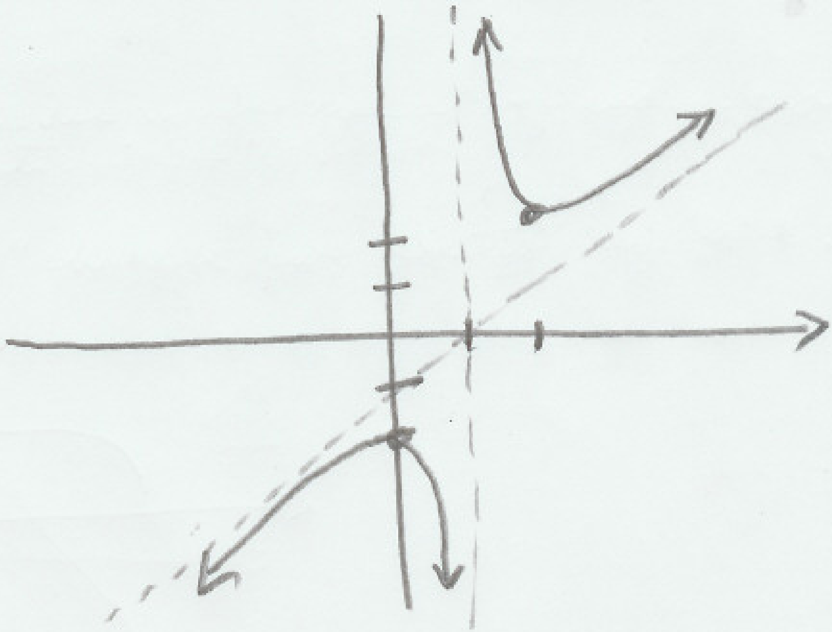
11. Analyze and sketch: $y = x - 1 + \dfrac{1}{x - 1}$



VA: $x = 1$

OA: $y = x - 1$

$y' = 1 - \dfrac{1}{(x-1)^2}$

$\boxed{x = 0, 2, 1}$

$0 = 1 - \dfrac{1}{(x-1)^2}$

$\dfrac{1}{(x-1)^2} = 1$

$(x-1)^2 = 1$

$x - 1 = \pm 1$

$x = 1 \pm 1$

$\boxed{x = 0, 2}$

$CV = 0, 2, 1$

y'	+	−	−	+
y	inc	dec	dec	inc
	0	1	2	

Rel Max at $(0, -2)$

Rel Min at $(2, 2)$

$y'' = \dfrac{2}{(x-1)^3}$

$x = 1$

y''	−	+
y	ccdn	ccup.
	1	