AP Calculus AB

SRP Answers

I. ① $\lim_{x\to -5} \frac{x-5}{x+5}$ d.n.e

$\lim_{x\to -5^+} \frac{x-5}{x+5} = \frac{-\text{large}}{+\text{small}} \Rightarrow -\infty$

$\lim_{x\to -5^-} \frac{x-5}{x+5} = \frac{-\text{large}}{-\text{small}} \Rightarrow \infty$

② $\lim_{x\to 2} \frac{2-x}{(x-2)^3} = \frac{0}{0}$

$\lim_{x\to 2} \frac{2-x}{(x-2)(x-2)^2}$

$\lim_{x\to 2} \frac{-1}{(x-2)^2}$ d.n.e

$\lim_{x\to 2^+} \frac{-1}{(x-2)^2} = -\infty \quad \lim_{x\to 2^-} \frac{-1}{(x-2)^2} = -\infty$

③ $\lim_{x\to -3} \frac{x+3}{x^3+8} = \frac{0}{-19} = \boxed{0}$

④ $\lim_{x\to 4} \frac{3\sqrt{x}-6}{x-4}$ $\frac{0}{0}$ so rationalize

$\lim_{x\to 4} \frac{3\sqrt{x}-6}{x-4} \cdot \frac{3\sqrt{x}+6}{3\sqrt{x}+6}$

$\lim_{x\to 4} \frac{9x-36}{(x-4)(3\sqrt{x}+6)} = \lim_{x\to 4} \frac{9(x-4)}{(x-4)(3\sqrt{x}+6)}$

$\lim_{x\to 4} \frac{9}{3\sqrt{x}+6} = \frac{9}{12} = \boxed{\frac{3}{4}}$

⑤ $\lim_{x\to 21} \frac{\frac{3}{x-7} - \frac{3}{14}}{x-21}$ $\frac{0}{0}$

$\lim_{x\to 21} \frac{\frac{3}{x-7} - \frac{3}{14}}{x-21} \cdot \frac{14(x-7)}{14(x-7)}$

$\lim_{x\to 21} \frac{42-3(x-7)}{14(x-21)(x-7)}$

$\lim_{x\to 21} \frac{63-3x}{14(x-21)(x-7)}$

$\lim_{x\to 21} \frac{3(21-x)}{14(x-21)(x-7)}$

$\lim_{x\to 21} \frac{-3}{14(x-7)} = \frac{-3}{14(14)} = \boxed{\frac{-3}{196}}$

⑥ $\lim_{x\to\infty} \frac{9x^3-3x+8}{5-2x-x^2+11x^3} = \boxed{\frac{9}{11}}$

⑦ $\lim_{x\to\infty} \frac{7x^2-8x+\pi}{5x-2} = \boxed{\infty}$

⑧ $\lim_{x\to\infty} \frac{6x+9x^2-x^3}{3x^2+1} = \boxed{-\infty}$

⑨ $\lim_{x\to -\infty} \frac{7x^3-6x^2+8}{9-x} = \boxed{-\infty}$

⑩ $\lim_{x\to 0} \frac{\cos x}{\cot x}$

$\lim_{x\to 0} \frac{\cos x}{\frac{\cos x}{\sin x}}$

$\lim_{x\to 0} \cos x \cdot \frac{\sin x}{\cos x}$

$\lim_{x\to 0} \sin x = \boxed{0}$

⑪ $\lim_{x\to -1} \tan^{-1} x$

$\tan^{-1}(-1)$

$\boxed{-\frac{\pi}{4}}$

⑫ $\lim_{x\to 3} \sec \frac{\pi x}{4}$

$\sec \frac{3\pi}{4}$

$\boxed{-\sqrt{2}}$

⑬ $\lim_{x\to \frac{\pi}{4}} \sec x$

$\sec \frac{\pi}{4}$

$\boxed{\sqrt{2}}$

⑭ $\lim_{x\to -5} \csc \frac{\pi x}{3}$

$\csc -\frac{5\pi}{3}$

$\boxed{\frac{2}{\sqrt{3}}}$

⑮ $\lim_{x\to -4} \frac{x+4}{x^2+16} = \boxed{0}$

⑯ $\lim_{x\to 0} \frac{4-4\cos x}{x}$

$\lim_{x\to 0} \frac{4(1-\cos x)}{x}$

$4 \cdot 0 = \boxed{0}$

⑰ $\lim_{x\to 0} \frac{\sin 5x}{x} \cdot \frac{5}{5}$

$\lim_{x\to 0} \frac{5\sin 5x}{5x}$

$5 \cdot 1 = \boxed{5}$

⑱ $\lim_{x\to 0} \frac{3x}{\pi \sin 8x} \cdot \frac{8}{8}$

$\frac{3}{\pi} \lim_{x\to 0} \frac{8x}{8\sin 8x}$

$\frac{3}{8\pi} \cdot 1 = \boxed{\frac{3}{8\pi}}$

(19) $\lim_{x\to 0} \frac{\tan x}{x}$

$\lim_{x\to 0} \frac{\sin x}{\cos x} \cdot \frac{1}{x}$

$\lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x}$

$1 \cdot 1 = \boxed{1}$

(20) $\lim_{x\to 2} \frac{x^3-1}{x-2}$ dne

$\lim_{x\to 2^+} \frac{x^3-1}{x-2} = \infty$ $\lim_{x\to 2^-} \frac{x^3-1}{x-2} = -\infty$

(21) $\lim_{x\to -67} [\![x]\!]$ dne

$\lim_{x\to -67^+} [\![x]\!] = -67$ $\lim_{x\to -67^-} [\![x]\!] = -68$

(22) $\lim_{x\to -3} \frac{|x+3|}{x+3}$ dne

$\lim_{x\to -3^+} \frac{|x+3|}{x+3} = 1$ $\lim_{x\to 3^-} \frac{|x+3|}{x+3} = -1$

II ① $f'(x) = \lim_{\Delta x\to 0} \frac{(x+\Delta x)^3 - (x+\Delta x) - (x^3-x)}{\Delta x}$

$= \lim_{\Delta x\to 0} \frac{x^3+3x^2\Delta x+3x\Delta x^2+\Delta x^3-x-\Delta x-x^3+x}{\Delta x}$

$= \lim_{\Delta x\to 0} \frac{3x^2\Delta x+3x\Delta x^2+\Delta x^3-\Delta x}{\Delta x}$

$= \lim_{\Delta x\to 0} 3x^2+3x\Delta x+\Delta x^2-1 = \boxed{3x^2-1}$

② $f'(x) = \lim_{\Delta x\to 0} \frac{\frac{-2}{(x+\Delta x)^3} - \frac{-2}{x^3}}{\Delta x}$

$= \lim_{\Delta x\to 0} \frac{-2x^3+2(x+\Delta x)^3}{\Delta x(x^3)(x+\Delta x)^3}$

$= \lim_{\Delta x\to 0} \frac{-2x^3+2x^3+6x^2\Delta x+6x\Delta x^2+2\Delta x^3}{\Delta x(x^3)(x+\Delta x)^3}$

$= \lim_{\Delta x\to 0} \frac{6x^2\Delta x+6x\Delta x^2+2\Delta x^3}{\Delta x(x^3)(x+\Delta x)^3}$

$= \lim_{\Delta x\to 0} \frac{6x^2+6x\Delta x+2\Delta x^2}{x^3(x+\Delta x)^3} = \boxed{\frac{6}{x^2}}$

③ $f'(x) = \lim_{\Delta x\to 0} \frac{2\sqrt{x+\Delta x+2} - 2\sqrt{x+2}}{\Delta x} \cdot \frac{2\sqrt{x+\Delta x+2}+2\sqrt{x+2}}{2\sqrt{x+\Delta x+2}+2\sqrt{x+2}}$

$= \lim_{\Delta x\to 0} \frac{4(x+\Delta x+2) - 4(x+2)}{\Delta x(2\sqrt{x+\Delta x+2}+2\sqrt{x+2})}$

$= \lim_{\Delta x\to 0} \frac{4\Delta x}{\Delta x(2\sqrt{x+\Delta x+2}+2\sqrt{x+2})}$

$= \lim_{\Delta x\to 0} \frac{4}{2\sqrt{x+\Delta x+2}+2\sqrt{x+2}} = \boxed{\frac{1}{\sqrt{x+2}}}$

III ① $f'(2) = \lim_{x\to 2} \frac{x^2-x-[(2)^2-2]}{x-2}$

$= \lim_{x\to 2} \frac{x^2-x-2}{x-2}$

$= \lim_{x\to 2} \frac{(x-2)(x+1)}{x-2} = \boxed{3}$

② $f'(x) = \lim_{x\to 1} \frac{\frac{1}{x} - \frac{1}{1}}{x-1}$

$= \lim_{x\to 1} \frac{1-x}{x(x-1)}$

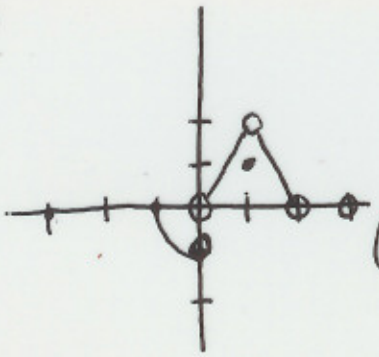
$= \lim_{x\to 1} \frac{-1}{x} = -1$

$f'(x) = \lim_{x\to 1} \frac{x-1}{x-1}$

$= 1$

derivative does not exist since left ≠ right

IV ①



② $f(x)$ is discontinuous at $x=0$ since:
(i) $f(0)$ is undefined

③ $f(x)$ is discont. at $x=1$ since:
(i) $f(1)=1$
(ii) $\lim_{x\to 1} f(x)=2$
(iii) $f(1)\neq \lim_{x\to 1}$

④ $f(x)$ is disc. @ $x=2$ since (i) $f(2)$ is undefined

⑤ disc. at $x=0$ non-removable
$x=1$ removable
$x=2$ non-removable

⑥ $f(x)$ is not differentiable since f is undefined at $x=2$

V $s(t)=-16t^2+128t$

① $s(0)=0$ ft

② $v(0)=128$ ft/sec

③ $v(t)=-32t+128$

④ $0=-32t+128$
$t=\frac{128}{32}=4$ ft

⑤ $a(t)=-32$, so never

⑥ $0=-16t^2+128t$
$0=-16t(t-8)$
$t=0,$ 8 sec

⑦ $v(8)=-32(8)+128=-128$ ft/sec

⑧ $\frac{s(7)-s(5)}{7-5}$
$\frac{112-240}{2}$
-64 ft/sec

⑨ The speed is increasing since $v(4.1)<0$ and $a(4.1)<0$

VI

① $y=x^{\frac{1}{3}}$
$y'=\frac{1}{3}x^{-\frac{2}{3}}=\frac{1}{3x^{2/3}}$

② $f(x)=x^{\frac{3}{5}}-4x^{\frac{1}{2}}-\frac{1}{x}$
$f'(x)=\frac{3}{5}x^{-\frac{2}{5}}-2x^{-\frac{1}{2}}+x^{-2}$
$=\frac{3}{5x^{2/5}}-\frac{2}{x^{\frac{1}{2}}}+\frac{1}{x^2}$

③ $f(x)=(x^2+2)^2=x^4+4x^2+4$
$f'(x)=4x^3+8x$

④ $f(x)=\frac{2x^5+8x^3+x}{x^3}$
$f(x)=2x^2+8+x^{-2}$
$f'(x)=4x-2x^{-3}$
$f'(x)=4x-\frac{2}{x^3}$

⑤ $f(x)=x^2+13x+\pi^3$
$f'(x)=2x+13$

⑥ $h(x)=x^{\frac{1}{3}}+x^{\frac{1}{5}}+x^{-\frac{1}{2}}$
$h'(x)=\frac{1}{3}x^{-\frac{2}{3}}+\frac{1}{5}x^{-\frac{4}{5}}-\frac{1}{2}x^{-\frac{3}{2}}$
$h'(x)=\frac{1}{3x^{\frac{2}{3}}}+\frac{1}{5x^{\frac{4}{5}}}-\frac{1}{2x^{3/2}}$

VII. ① $f(x)=6x-2x^2-7x^3$

$f'(x)=6-4x-21x^2$

$f'(-1)=6-4(-1)-21(-1)^2=-11$

$f(-1)=-1$

$\boxed{y+1=-11(x+1)}$

② $y=x^{\frac{1}{2}}+5x$

$y'=\frac{1}{2}x^{-\frac{1}{2}}+5$

$y'(4)=\frac{1}{2\sqrt{4}}+5=\frac{21}{4}$

$y(4)=22$

$y-22=-\frac{4}{21}(x-4)$

③ $f(x)=x^3$

$f'(x)=3x^2$

$\frac{4}{3}=3x^2$

$4=9x^2$

$\frac{4}{9}=x^2$

$\pm\frac{2}{3}=x$

$f(\frac{2}{3})=\frac{8}{27}$

$f(-\frac{2}{3})=-\frac{8}{27}$

$y-\frac{8}{27}=\frac{4}{3}(x-\frac{2}{3})$

$y+\frac{8}{27}=\frac{4}{3}(x+\frac{2}{3})$

VIII ① 6 ② 2 ③ undefined

④ $-\infty$ ⑤ 0 ⑥ 1 ⑦ 1

⑧ Decr: $(0,3)(5,\infty)$

Incr: $(-\infty,-3)(-3,0)(3,5)$

⑨ D: $(-\infty,-3)(-3,\infty)$

R: $(-\infty,7]$

⑩ $(-\infty,-3)(-3,3)(3,\infty)$

IX ① $4\sin 6x\cos 6x$

$2\sin 12x$

② $25(1-2\sin^2 10\beta)$

$25\sin 20\beta$

③ $\sin 3x\cos 3x$

$\frac{1}{2}\sin 6x$

④ $\frac{1}{2}\sin\theta\cos\theta$

$\frac{1}{4}\sin 2\theta$

⑤ $\pi(\cos^2\pi x-\sin^2\pi x)$

$\pi\cos 2\pi x$

⑥ $50(2\cos^2 50\alpha-1)$

$50\cos 100\alpha$

⑦ $\frac{2\tan x}{1-\tan^2 x}$

$\tan 2x$

XI $\frac{(x^2+4)^{-\frac{2}{3}}\left[(x^2+4)-\frac{2}{3}x^2\right]}{(x^2+4)^{\frac{4}{3}}}$

$\frac{x^2+4-\frac{2}{3}x^2}{(x^2+4)^{\frac{5}{3}}}$

$\frac{\frac{1}{3}x^2+4}{(x^2+4)^{\frac{5}{3}}}$

XII ① $-\frac{1}{2}$ ② 2 ③ -1 ④ 2 ⑤ -1.452

⑥ 9.109 ⑦ $-\frac{\pi}{2}$ ⑧ 1.123 ⑨ π ⑩ .227

XIII ① $\tan x=.2318$

$\tan^{-1}.2318=x$

$x=.228$

② $\sin x=-.5057$

$\sin^{-1}-.5057=x$

$x=-.530$

③ $\cos^{-1}-2.6198$

$\Rightarrow$ no solution