



AP Calculus AB 2001 Free-Response Questions

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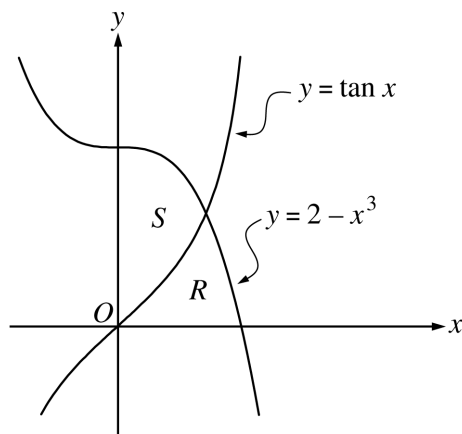
2001 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



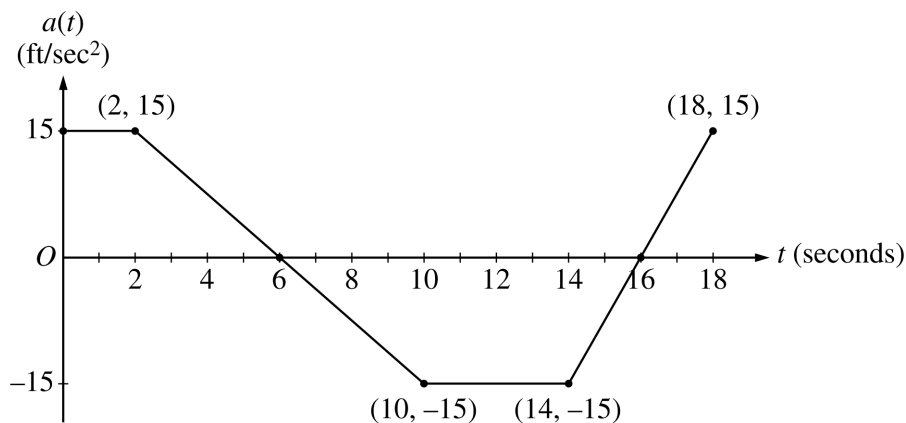
- Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x -axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y -axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.
 - Find the area of R .
 - Find the area of S .
 - Find the volume of the solid generated when S is revolved about the x -axis.
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t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

2. The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t . The table above shows the water temperature as recorded every 3 days over a 15-day period.
- (a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.
- (b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \leq t \leq 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.
- (c) A student proposes the function P , given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t , where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.
- (d) Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \leq t \leq 15$ days.
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3. A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \leq t \leq 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2 , is the piecewise linear function defined by the graph above.
- (a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?
 - (b) At what time in the interval $0 \leq t \leq 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?
 - (c) On the time interval $0 \leq t \leq 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.
 - (d) At what times in the interval $0 \leq t \leq 18$, if any, is the car's velocity equal to zero? Justify your answer.
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END OF PART A OF SECTION II

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CALCULUS AB SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

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4. Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by

$$h'(x) = \frac{x^2 - 2}{x} \text{ for all } x \neq 0.$$

- (a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.
- (b) On what intervals, if any, is the graph of h concave up? Justify your answer.
- (c) Write an equation for the line tangent to the graph of h at $x = 4$.
- (d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

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5. A cubic polynomial function f is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

where a , b , and k are constants. The function f has a local minimum at $x = -1$, and the graph of f has a point of inflection at $x = -2$.

- (a) Find the values of a and b .
- (b) If $\int_0^1 f(x) dx = 32$, what is the value of k ?

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6. The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6 - 2x)$.

- (a) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

- (b) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6 - 2x)$ with the initial condition $f(3) = \frac{1}{4}$.

END OF EXAMINATION