



AP[®] Calculus AB 1998 Free-Response Questions

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1998 AP Calculus AB Free-Response Questions

CALCULUS AB

Section II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUCTIONS.

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1. Let R be the region bounded by the x -axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.
- (a) Find the area of the region R .
 - (b) Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.
 - (c) Find the volume of the solid generated when R is revolved about the x -axis.
 - (d) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x -axis, they generate solids with equal volumes. Find the value of k .
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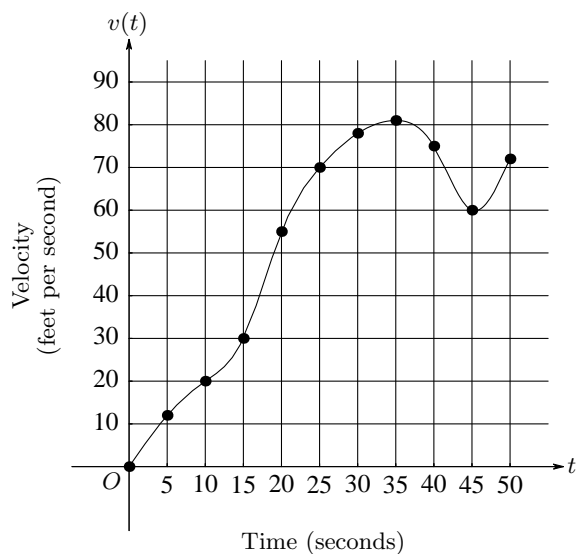
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2. Let f be the function given by $f(x) = 2xe^{2x}$.
- (a) Find $\lim_{x \rightarrow -\infty} f(x)$ and $\lim_{x \rightarrow \infty} f(x)$.
 - (b) Find the absolute minimum value of f . Justify that your answer is an absolute minimum.
 - (c) What is the range of f ?
 - (d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b .
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t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

3. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \leq t \leq 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t , is shown to the right of the graph.
- During what intervals of time is the acceleration of the car positive? Give a reason for your answer.
 - Find the average acceleration of the car, in ft/sec², over the interval $0 \leq t \leq 50$.
 - Find one approximation for the acceleration of the car, in ft/sec², at $t = 40$. Show the computations you used to arrive at your answer.
 - Approximate $\int_0^{50} v(t) dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

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4. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\frac{3x^2 + 1}{2y}$.
- (a) Find the slope of the graph of f at the point where $x = 1$.
 - (b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.
 - (c) Find $f(x)$ by solving the separable differential equation $\frac{dy}{dx} = \frac{3x^2 + 1}{2y}$ with the initial condition $f(1) = 4$.
 - (d) Use your solution from part (c) to find $f(1.2)$.
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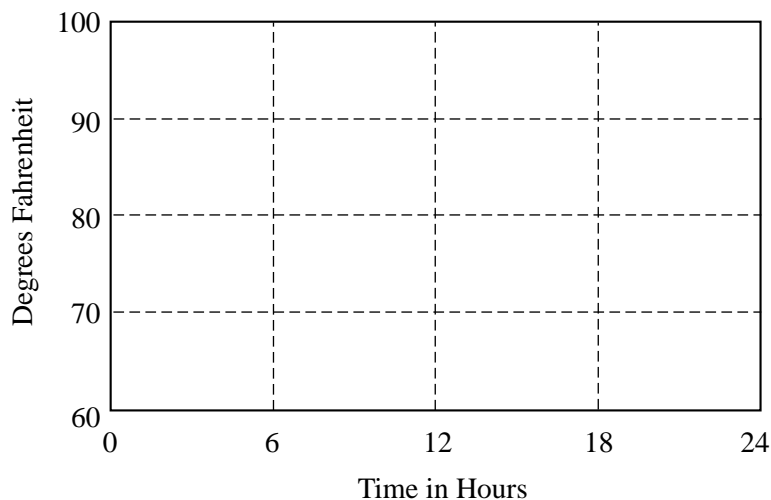
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5. The temperature outside a house during a 24-hour period is given by

$$F(t) = 80 - 10 \cos\left(\frac{\pi t}{12}\right), \quad 0 \leq t \leq 24,$$

where $F(t)$ is measured in degrees Fahrenheit and t is measured in hours.

- (a) Sketch the graph of F on the grid below.



- (b) Find the average temperature, to the nearest degree Fahrenheit, between $t = 6$ and $t = 14$.
- (c) An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of t was the air conditioner cooling the house?
- (d) The cost of cooling the house accumulates at the rate of \$0.05 per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24-hour period?
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6. Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.

(a) Show that $\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + y^2 + 1}$.

(b) Write an equation of each horizontal tangent line to the curve.

(c) The line through the origin with slope -1 is tangent to the curve at point P . Find the x - and y -coordinates of point P .

END OF EXAMINATION