AP® Calculus AB
1998 Free-Response Questions

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CALCULUS AB

Section II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUCTIONS.

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1. Let R be the region bounded by the x–axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.
   (a) Find the area of the region R.
   (b) Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.
   (c) Find the volume of the solid generated when R is revolved about the x–axis.
   (d) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x–axis, they generate solids with equal volumes. Find the value of k.

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2. Let f be the function given by $f(x) = 2xe^{2x}$.

   (a) Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$.

   (b) Find the absolute minimum value of f. Justify that your answer is an absolute minimum.

   (c) What is the range of f?

   (d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

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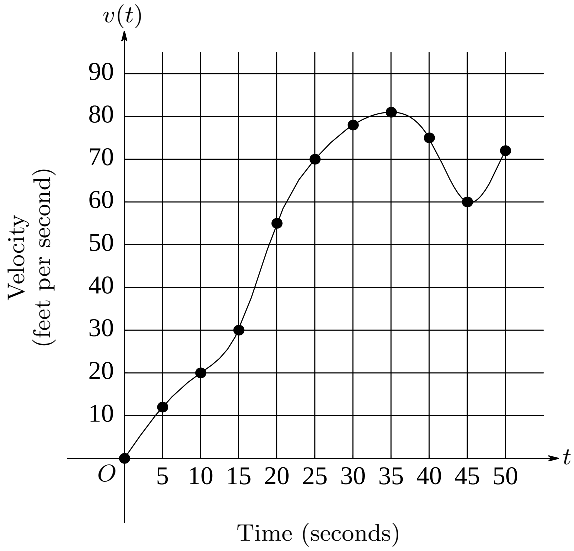

t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

3. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.

   (a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

   (b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \le t \le 50$.

   (c) Find one approximation for the acceleration of the car, in ft/sec^2, at $t = 40$. Show the computations you used to arrive at your answer.

   (d) Approximate $\int_0^{50} v(t)\,dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

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4. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\dfrac{3x^2 + 1}{2y}$.

   (a) Find the slope of the graph of f at the point where $x = 1$.

   (b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.

   (c) Find $f(x)$ by solving the separable differential equation $\dfrac{dy}{dx} = \dfrac{3x^2 + 1}{2y}$ with the initial condition $f(1) = 4$.

   (d) Use your solution from part (c) to find $f(1.2)$.

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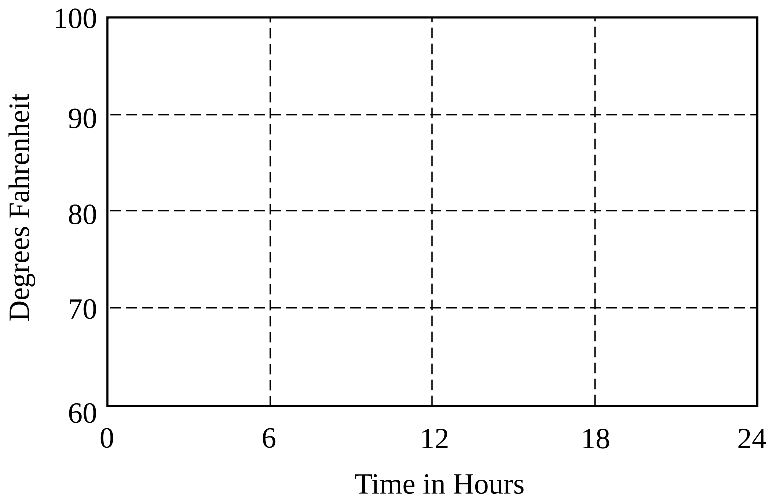
5. The temperature outside a house during a 24-hour period is given by

$$F(t) = 80 - 10\cos\left(\frac{\pi t}{12}\right),\ 0 \le t \le 24,$$

where $F(t)$ is measured in degrees Fahrenheit and t is measured in hours.

(a) Sketch the graph of F on the grid below.



(b) Find the average temperature, to the nearest degree Fahrenheit, between $t = 6$ and $t = 14$.

(c) An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of t was the air conditioner cooling the house?

(d) The cost of cooling the house accumulates at the rate of \$0.05 per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24–hour period?

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6. Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.

   (a) Show that $\dfrac{dy}{dx} = \dfrac{4x - 2xy}{x^2 + y^2 + 1}$.

   (b) Write an equation of each horizontal tangent line to the curve.

   (c) The line through the origin with slope -1 is tangent to the curve at point P. Find the x– and y–coordinates of point P.

END OF EXAMINATION